



## INVESTIGATING STYLIZED FACTS OF ASSET RETURNS IN AN EMERGING STOCK MARKET IN THE PRESENCE OF GAUSSIAN ERRORS



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**Abstract:** This paper models the volatility of asset returns that produces several well-documented stylized facts. The Guinness bottling Company Plc daily closing share prices of the Nigerian stock exchange is used as proxy for Nigerian stock market. The data used for the study covers the period 1/02/1995 to 24/11/2014. The study employed Augmented Dickey-Fuller (ADF) unit root test for stationarity and mean reverting properties of returns while the volatility of asset returns was modeled using symmetric standard GARCH (1,1), asymmetric TGARCH (1,1) and PGARCH (1,1) models with Gaussian errors. Results of the unit root test indicate that the returns are stationary and mean reverting. The standard GARCH (1,1) model showed evidence of volatility clustering and mean reversion in Nigerian stock market. The conditional volatility was found to be quite persistence. The estimated asymmetric TGARCH (1,1) and PGARCH (1,1) models produced supportive evidence to the existence of asymmetry and leverage effects in Nigerian stock market. The study also found that the log returns are non-Gaussian, leptokurtic, fat-tailed and serially uncorrelated. The study therefore concludes that the well-documented stylized facts found in advanced and developed markets are also present in emerging stock markets like Nigeria and recommends that both local and foreign traders and investors should invest heavily in Guinness Plc as it has stable and mean reverting asset returns which are less risky.

**Keywords:** Asset returns, GARCH models, Gaussianity, heteroskedasticity, stylized facts, volatility, Nigeria

### Introduction

Volatility modeling of financial time series data such as asset returns has become an interesting area of research among researchers and practitioners. This is because volatility is an important concept for many economic and financial applications such as portfolio optimization, risk management, options trading and asset pricing. According to Tsay (2002), a special feature of volatility which is the conditional variance of the underlying asset returns, is that it is not directly observable. Thus, financial analysts have keen interest in obtaining good estimates of this conditional variance in order to improve portfolio allocation, risk management and valuation of financial derivatives. Various types of models such as autoregressive conditional heteroskedasticity and stochastic volatility models have been applied in modeling volatility.

In modeling the volatility of financial time series data researchers have revealed a wealth of interesting statistical properties called “stylized facts”. Stylized facts are empirical findings that are so consistent and believed to hold for a diverse collection of instruments, markets and time periods. A number of researchers have studied stylized facts of asset returns including studies by Cont (2001); Ding *et al.* (1983); Guillaume *et al.* (1997) and Pagan (1986) who summarized the most important stylized facts of assets returns as: Absence of autocorrelations, non Gaussianity, heavy/fat tails, aggregational Gaussianity, gain/loss asymmetry, leverage effect, volatility clustering, and volatility mean reversion among others. A good volatility model must then be able to capture and reflect these stylized facts.

The first documented evidence of volatility clustering, leptokurtosis and leverage effects was observed by Mandelbrot (1963) who found evidence of the tendency of large changes in asset prices (either positive or negative) to be followed by large changes in asset prices and small changes in asset prices to be followed by small changes. Similar results were found in studies conducted independently by Fama (1965) and Black (1976). A vast of documented evidence on the subject matter both for developed and emerging stock markets are found in the literature. See for example; Harris (1986), Fama (1970), Du and Ning (2008), Fama and French (1988), Ding and Granger (1996), Granger and Ding (1996), Gibbons and Hess (1981), Granger and Hyung (2004), Greene

& Fielitz (1977), Hamao & Hasbrouck (1995), Hansen & Lunde (2006) among others for more surveys.

In Nigeria, several studies have been conducted on volatility modelling which provide more insights on the subject matter. For instance, Olowe (2009) investigated the relationship between stock returns and volatility in Nigeria using EGARCH-in-mean model in the light of banking reforms, insurance reform, stock market crash and the global financial crisis. He used daily returns for the period 4 January 2004 to January 9, 2009. The result shows persistence of volatility and presence of leverage effects. In a related development Okpara (2011) conducted a study to investigate the relationship between the stock market returns and volatility in Nigerian stock market using the same EGARCH-in-mean framework. He used monthly stock price data from Nigerian stock Exchange. The study found the Nigerian stock market to be volatile with high level of risk in stock trading with leverage asymmetric effects. The study also found low persistence of volatility clustering which suggest that increase in volatility is not likely to remain high over several periods. Awogbemi & Alagbe (2011) examined the volatility of Naira/US Dollar and Naira/UK Pound Sterling exchange rates in Nigeria using GARCH model. They used data on monthly exchange rates for the period 2007-2010 and found evidence of volatility persistence and clustering in Nigeria.

Onakoya (2013) conducted a study to examine the relative contributions of stock market volatility on economic growth in Nigeria for the period of 1980 to 2010 using Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model. The study found volatility shocks to be quite persistent in Nigeria and remarked that this might distort growth of the economy. Adesina (2013) used symmetric and asymmetric GARCH models to estimate stock return volatility in Nigerian Stock Exchange (NSE). The study used 324 monthly data from January 1985 to December 2011 of the NSE all share-index and found enough evidence of volatility clustering and high persistence of volatility for the NSE return series with no asymmetric shock phenomenon (leverage effects) for the return series. Olusola & Opeyemi (2013) used a parametric measure to study the trend and possible causes of exchange rate volatility in Nigeria for the period 1986:1 to 2009:4. The study revealed that exchange rate has been volatile in Nigeria which portrays higher risk to a risk-averse

economic agent. On investigating the nature of volatility clustering, persistence and leptokurtic nature of asset returns in Nigeria using individual bank indices and the All-share Index of the Nigerian Stock Exchange, Emenike & Ani (2014) found evidence of volatility clustering, persistence and fat tail distribution which have insignificant influence on the volatility of stock returns of the banks. Osazevaru (2014a) empirically examined the presence or otherwise of volatility clustering in Nigerian stock market using time series data of daily share prices for the period 1995 to 2009. He employed the Autoregressive Conditional Heteroscedasticity (ARCH) Model and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The estimates indicate that the market exhibits volatility clustering and the rate at which the response function decays was found to be very high. He suggested that aggressive trading on a wide range of securities be encouraged as this will increase market depth and hence reduce volatility.

Osazevaru (2014b) investigated the hypothesized relationship between market news and volatility using daily and monthly stock data of the Nigerian stock market for the period 1995 to 2011. He used Threshold Generalized Autoregressive Conditional Heteroscedasticity, TGARCH (1,1) model and found no asymmetries in the market news and the impact of bad news was not larger on volatility than good news. He also found the Nigerian market to be such that old information yields more importance than recent information. Uwubanmwun & Omorokunwa (2015) also found evidence of volatility clustering and persistence in Nigeria by showing that oil price volatility generates and stimulates stock prices volatility in Nigeria.

From the above, it glaring that while independent researchers used different volatility models across different economies to investigate stylized facts of financial returns all agreed that some of these empirical properties exist. Although in Nigeria, most researchers are interested in examining volatility clustering, persistence, leptokurtosis and leverage effects. This study investigates the stylized facts characterized by developed markets in emerging stock markets like Nigeria using GARCH invariants and more recent data. The parameter estimation procedure adopted in this study assumes Gaussian error structure typical of GARCH-type models unlike the ones found in the literature which assumed non-Gaussian errors.

**Materials and Methods**

**Data for the study**

The data used in this study are the daily closing share prices of Guinness Plc taken from Nigerian Stock Exchange (NSE) website ([www.nse.com](http://www.nse.com)). The time series data covers the period of 19 years from 1<sup>st</sup> February, 1995 to 31<sup>st</sup> December, 2014 making a total of 4927 observations. The share prices are in Nigerian naira. The daily returns  $r_t$  are calculated as the continuously compounded returns corresponding to the first difference in logarithms of closing prices of successive days as:

$$r_t = \ln\left(\frac{R_t}{R_{t-1}}\right) \times 100 = [\ln(R_t) - \ln(R_{t-1})] \times 100 \quad (1)$$

where  $R_t$  denotes the closing market index at the current day ( $t$ ) and  $R_{t-1}$  denotes the closing market index at the previous day ( $t - 1$ ).

**Test of normality**

Jarque and Bera (1980, 1987) proposed a normality test, which provides a goodness-of-fit test on whether sample data have the skewness and kurtosis matching a normal distribution. For the return series  $\{r_t\}$  under study, the test statistic JB is defined as:

$$JB = \frac{T}{6} \left( S_k^2 + \frac{1}{4} (K_u - 3)^2 \right) \quad (2)$$

where  $T$  is the number of observations,  $S_k$  is the sample skewness which is estimated by:

$$S_k = \frac{\mu_3}{\mu_2^{3/2}} = T^{1/2} \frac{\sum_{t=1}^T (r_t - \bar{r})^3}{\left( \sum_{t=1}^T (r_t - \bar{r})^2 \right)^{3/2}} ;$$

and  $K_u$  is the sample kurtosis which is estimated by:

$$K_u = \frac{\mu_4}{\mu_2^2} = T \frac{\sum_{t=1}^T (r_t - \bar{r})^4}{\left( \sum_{t=1}^T (r_t - \bar{r})^2 \right)^2}$$

(see Alexander, 2008).

Volatility refers is associated with the standard deviation  $\sigma$  of returns over some period of time. It is calculated from sample observation as

$$\hat{\sigma} = \sqrt{\frac{1}{T-1} \sum_{t=1}^T [r_t - u]^2} \quad (3)$$

where  $r_t$  is the return of an asset over period  $t$  and  $u$  is the average return over  $T$  periods.

**Unit root tests**

To check for the presence of unit root in the returns, we employ Augmented Dickey-Fuller unit root test. Said and Dickey (1984) augment the basic autoregressive unit root test to accommodate general ARMA (p,q) models with unknown orders and their test is referred to as the augmented Dickey-Fuller (ADF) test. The ADF test tests the null hypothesis that a time series  $y_t$  is I(1) against the alternative that it is I(0), assuming that the dynamics in the data have an ARMA structure. The ADF test is based on estimating the test regression:

$$Y_t = \beta' D_t + \phi Y_{t-1} + \sum_{j=1}^p \psi_j \Delta Y_{t-j} + \varepsilon_t \quad (4)$$

where  $D_t$  is a vector of deterministic terms (constant, trend etc.). The  $p$  lagged difference terms,  $\Delta y_{t-j}$ , are used to approximate the ARMA structure of the errors, and the value of  $p$  is set so that the error  $\varepsilon_t$  is serially uncorrelated. The error term is also assumed to be homoskedastic. The specification of the deterministic terms depends on the assumed behaviour of  $y_t$  under the alternative hypothesis of trend stationarity. Under the null hypothesis,  $y_t$  is I(1) which implies that  $\phi = 1$ . The ADF  $t$ -statistic and normalized bias statistic are based on the least squares estimates of equation (4) and are given by:

$$ADF_t = t_{\phi-1} = \frac{\hat{\phi} - 1}{SE(\hat{\phi})} = \frac{\sum_{t=2}^n R_{t-1} e_t}{\hat{\sigma}^2 \sqrt{\sum_{t=2}^n R_{t-1}^2}} \quad (5)$$

$$ADF_n = \frac{T(\hat{\phi} - 1)}{1 - \hat{\psi}_1 - \hat{\psi}_2 - \dots - \hat{\psi}_p} \quad (6)$$

The null hypothesis is rejected if the calculated value of  $t$  is greater than  $t$  critical.

In choosing the lag length  $p$  for the ADF test, a useful rule of thumb for determining an upper bound for  $p$  suggested by Schwert (1989) is:

$$p_{max} = \left\lceil 12 \cdot \left( \frac{T}{100} \right)^{1/4} \right\rceil \quad (7)$$

where  $p_{max}$  is an upper bound for  $p$  and  $\lceil x \rceil$  denotes the integer part of  $x$ . This choice allows  $p_{max}$  to be growing with the sample so that the ADF test regression (4) is valid if the errors follow an ARMA process with unknown order.

**Test for ARCH effects**

To test for ARCH effects (Heteroscedasticity) the Lagrangian Multiplier Test of Engle is used. The null hypothesis is  $H_0: \alpha_1 = \dots = \alpha_m$  versus  $H_1: \alpha_1 \neq 0$  for some  $i \in \{1, \dots, m\}$ .

The F-statistic is then computed as:

$$F = \frac{SSR_0 - SSR_1/m}{SSR_1(n - 2m - 1)} \quad (8)$$

where  $SSR_1$

$$= \sum_{t=m+1}^T e_t^2, \hat{e}_t \text{ is the least square residual of the linear regression}$$

$$SSR_0 = \sum_{t=m+1}^T (a_t^2 - \bar{a})^2 \text{ and } \bar{a} = \frac{1}{n} \sum_{t=1}^T a_t^2 \text{ is the sample mean of } a_t^2.$$

The test statistic is asymptotically distributed as chi-square distribution with  $m$  degrees of freedom under the null hypothesis. The decision is to reject the null hypothesis if  $F > \chi_m^2(\alpha)$ , where  $\chi_m^2(\alpha)$  is the upper  $100(1 - \alpha)^{th}$  of the  $\chi_m^2$  or the p-value of F-statistic less than 5%.

**Model Specifications**

**The generalized ARCH (GARCH) models**

The ARCH model of Engle (1982) was generalized to GARCH model by Bollerslev (1986). GARCH model generalizes the ARCH model in the same way an ARMA model generalizes an MA model. The GARCH (p, q) model can be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (9)$$

with the constraints  $\omega > 0, \alpha_i \geq 0, i = 1, \dots, q$  and  $\beta_j \geq 0, j = 1, \dots, p$ . The standard GARCH (1,1) model is given by:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (10)$$

The stationarity condition for a standard GARCH (1,1) model is that the sum of ARCH and GARCH terms is strictly less than unity (i.e.,  $(\alpha_1 + \beta_1) < 1$ ). To ensure that the conditional variance  $\sigma_t^2$  is positive, the constraints  $\omega > 0, \alpha_1 \geq 0, \beta_1 \geq 0$  must be satisfied. If  $(\alpha_1 + \beta_1) > 1$ , the conditional variance process is non-stationary, unstable and therefore explodes.

**TGARCH(1,1) model**

The Threshold-GARCH (TGARCH) model was introduced by Zakoian (1994) and Glosten *et al.* (1993) to detect asymmetry in financial time series data. The generalized specification of the conditional variance for the TGARCH (1,1) model is as follows:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1} S_{t-1}^- \quad (11)$$

where  $S_{t-1}^-$  is a dummy variable, that is,  $S_t^- = 1$  if  $\varepsilon_t < 0$  and 0 if  $\varepsilon_t \geq 0$ . The coefficient  $\gamma$  in the model captures the asymmetric effect if  $\gamma > 0$ . The  $\omega, \alpha_1$  and  $\beta_1$  are the parameters of the conditional variance equation that are to be estimated.

In TGARCH (1,1) model, good news ( $\varepsilon_t \geq 0$ ) and bad news ( $\varepsilon_t < 0$ ) have different effects on the conditional variance; good news has an impact of  $\alpha_1$ , while bad news has an impact of  $\alpha_1 + \gamma$ . If  $\gamma > 0$ , bad news increases volatility, and we say that there is a leverage effect. If  $\gamma \neq 0$  the news impact is asymmetric. The condition for accepting the null hypothesis of no leverage effect in TGARCH model is that the coefficient of the parameter  $\gamma$  must be negative and insignificant. But if the  $\gamma$  coefficient is non-negative and significant, then there is evidence of leverage effects in the series.

**The power GARCH (PGARCH) model**

The Power ARCH (PARCH) model was first introduced by Taylor (1986) and Schwert (1989) where the standard deviation is modeled rather than the variance. This model

along with several other models is generalized in Ding *et al.* (1993) with the power GARCH (PGARCH) specification. In PGARCH model, the power parameter  $\delta$  of the standard deviation can be estimated rather than imposed, and the optional  $\gamma$  parameters are added to capture asymmetry of up or order  $r$ . The specification for PGARCH (1,1) in the conditional variance is given by:

$$\sigma_t^2 = \omega + \alpha_1 (|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta \quad (12)$$

In PGARCH (1,1) model the restrictions for the positivity of  $\sigma_t^\delta$  are given by Ding *et al.* (1993) as  $\omega > 0, \delta \geq 0, -1 < \gamma < 1, \alpha_1 \geq 0$  and  $\beta_1 \geq 0$ .

**Model Selection Criteria**

Akaike Information Criteria (AIC) due to (Akaike, 1974), Schwarz Information Criterion (SIC) due to (Schwarz, 1978) and Hannan-Quinn Information Criterion (HQC) due to (Hannan, 1980) and Log likelihood are the most commonly used model selection criteria. These criteria were used in this study and are computed as follows:

$$AIC(K) = -2 \ln(L) + 2K \quad (13)$$

$$SIC(K) = -2 \ln(L) + K \ln(T) \quad (14)$$

$$HQC(K) = 2 \ln[\ln(T)] K - 2 \ln(L) \quad (15)$$

where  $K$  is the number of independently estimated parameters in the model,  $T$  is the number of observations;  $L$  is the maximized value of the Log- Likelihood for the estimated model and is defined by:

$$L = \prod_{i=1}^n \left( \frac{1}{2\pi\sigma_i^2} \right)^{1/2} \exp \left[ - \sum_{i=1}^n \frac{(y_i - f(x))^2}{2\sigma_i^2} \right]$$

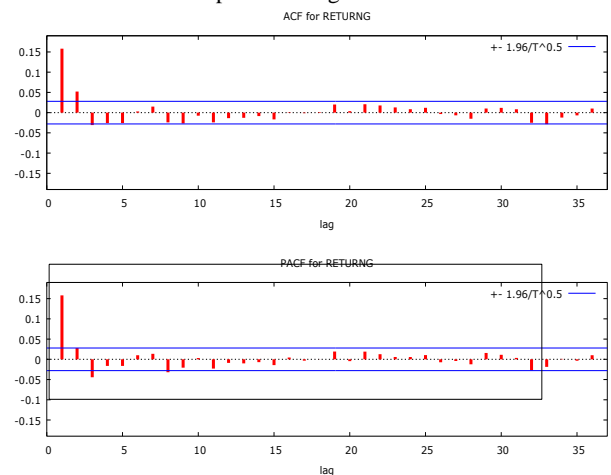
$$\text{Log } L = \ln \left[ \prod_{i=1}^n \left( \frac{1}{2\pi\sigma_i^2} \right)^{1/2} - \frac{1}{2} \sum_{i=1}^n \frac{(y_i - f(x))^2}{\sigma_i^2} \right] \quad (16)$$

Thus given a set of estimated GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and larger log likelihood value.

**Results and Discussion**

**Autocorrelation and partial autocorrelation functions**

Results of autocorrelation function (ACF) and partial autocorrelation function (PACF) of returns are examined here to identify degree of serial correlation in the data points of the series. The result is reported in Fig. 1.



**Fig. 1:** ACF and PACF of log returns

The ACF and PACF results show that the returns are serially correlated. This means that there is substantial dependence in the volatility of the returns series. Ljung-Box Q-statistic for serial correlation test reported in Table 1 shows that the autocorrelations of absolute returns are statistically insignificant while the autocorrelation function of log returns is statistically significant. This shows that absolute returns are

serially correlated whereas the log returns are not. This result satisfied one of the stylized facts of asset returns found in developed markets.

**Table 1: Ljung-Box Q-statistic test for serial correlation**

Lag	Absolute returns		Log returns	
	Q-statistic	P-value	Q-statistic	P-value
1	38.202	0.000*	0.0052	0.942
2	50.188	0.000*	0.0195	0.990
3	52.575	0.000*	0.8307	0.842
4	55.026	0.000*	1.3142	0.859
5	57.433	0.000*	1.5662	0.905
6	61.011	0.000*	2.0536	0.915
7	63.358	0.000*	2.1127	0.953
8	63.972	0.000*	2.2346	0.973
9	64.095	0.000*	2.2363	0.987
10	66.686	0.000*	2.5712	0.990
11	66.712	0.000*	2.6230	0.995
12	66.724	0.000*	2.6400	0.998

**Note:** \* denotes significant of Q-statistics at 1% marginal significance level.

**Result of Jarque-Bera test for normality**

The summary statistics for the return series are presented in Table 2.

**Table 2: Summary Statistics of Log Returns**

N	Mean	Max.	Min.	S.dev.	Var.	Skew.	Kurt.	JB	P-value
4927	0.062	9.844	-61.50	2.298	5.28	-4.29	111.87	2448323	0.0000

The results reveals that daily average returns is 0.062% which is positive, the daily standard deviation of returns is 2.298% with a variance of 5.28%. This indicates a high level of dispersion from the daily average returns in the market for the period under review. The wide gap between maximum and minimum returns gives support for the high level of variability of price changes in Nigerian stock market. The skewness coefficient indicates that the distribution of the returns is substantially negatively skewed; which is a common feature of financial asset returns. The kurtosis coefficient, which is a measure of the thickness of the tail of the distribution of returns exhibit high kurtosis (>3 of the normal distribution). This shows evidence of leptokurtosis, one of the stylized facts known in the early days of volatility modelling. Judging from skewness and kurtosis perspective, the returns are not normally distributed. The significant p-value of JB test as reported in Table 2 indicates that the null hypothesis of normality is rejected which implies that the returns are not normally distributed. This also satisfied the non-Gaussianity property of asset returns.

**ADF Unit root and heteroskedasticity test results**

Result of the unit root test is presented in Table 2.

The ADF unit root test results show that the daily share prices are non-stationary while the daily returns are stationary. This means that the returns are free from the presence of unit root and we thus reject the null hypothesis of the presence of unit root in returns.

**Table 2: ADF unit root test results**

Variable	Option	ADF Test Statistic	P-values	Critical Values		
				1%	5%	10%
Daily Share	Intercept only	-1.4120	0.5780	-3.4315	-2.8619	-2.5670
Prices ( $Y_t$ )	Intercept & Trend	-2.3660	0.6561	-3.9599	-3.4107	-3.1271
Daily Returns ( $r_t$ )	Intercept only	-59.8293	0.0001*	-3.4315	-2.8619	-2.5670
	Intercept & Trend	-59.8332	0.0000*	-3.9599	-3.4107	-3.1271

**Note:** \* denotes significant of ADF test statistics at 1% significance level.

The heteroskedasticity test for the presence of ARCH effect in returns, the Engle’s LM ARCH test is employed with result presented in Table 3.

The LM ARCH test confirms that the return series exhibits the presence of ARCH effect up to lag 31 corresponding to 6 trading weeks since the p-values of both F-statistics and  $nR^2$  are strictly less than 0.05 significance level. This means that the variance of returns is non-constant and can only be modeled using the family of GARCH models.

**Table 3: Heteroskedasticity test of asset returns**

Lag Order	F-statistic	P-value	$nR^2$	P-value
1	6.360034	0.0012	6.354408	0.0011
31	4.720912	0.0003	4.832901	0.0004

**Table 4: Estimate of symmetric GARCH (1,1) model with Gaussian errors**

Variable	Coefficient	Std. Error	z-Statistic	P-value
$\mu$	0.057566	0.021561	2.669920	0.0076
$\omega$	0.335410	0.012333	27.19655	0.0000
$\alpha_1$	0.268161	0.006581	40.74488	0.0000
$\beta_1$	0.706928	0.007218	97.94590	0.0000
$\alpha_1 + \beta_1$	0.975089			

**Symmetric GARCH (1,1)**

To examine whether there is volatility clustering and persistence in the Nigerian stock market, the result of GARCH (1,1) model under normal distribution is shown in Table 4.

The estimates of the symmetric GARCH (1,1) model shows that, in the conditional variance equation, the coefficient of the ARCH term ( $\alpha_1 = 0.268161$ ) is positive and statistically significant at 1% level showing that past news on volatility have explanatory power on the current volatility. The coefficient of GARCH term ( $\beta_1 = 0.706928$ ) is also positive and statistically significant at 1% level, showing that past volatility of stock market returns is significant and influences current volatility. The sum of ARCH and GARCH coefficients ( $\alpha_1 + \beta_1 = 0.975089$ ) which is a measure of volatility persistence, is close to unity and this implies that the volatility is significantly quite persistence in Nigerian stock market. The result of GARCH (1,1) thus indicates that memory of shocks is remembered in the NSE.

**Asymmetric TGARCH (1,1) and PGARCH (1,1) models**

To investigate the presence of asymmetry and leverage effects in the return series, two asymmetric GARCH models have been estimated namely: TGARCH (1,1) and PGARCH (1,1). The result is presented in Table 5.

The estimates of TGARCH (1,1) are shown in the upper panel of Table 5 while the estimates for PGARCH (1,1) are shown in the lower panel of Table 5. The asymmetric effect parameters  $\gamma$  captured by TGARCH (1,1) and PGARCH (1,1) are both positive and significant as expected thereby providing supportive evidence for the existence of asymmetric and leverage effects in the returns during the study period. This implies that previous period’s positive and negative shocks have different effects on the conditional variance. This also means that there is a tendency for changes in stock prices to be negatively correlated with changes in volatility. This

result is consistent with the findings of Olowe (2009) and Okpara (2011) who also found asymmetry and leverage effects in Nigerian Stock Market.

**Table 5: Estimates of asymmetric GARCH (1,1) with Gaussian errors**

Model	Variable	Coefficient	Std. Error	z-Statistic	P-value
TGARCH (1,1)**	$\mu$	0.0709	0.0157	4.5239	0.0000
	$\omega$	0.3472	0.0129	26.985	0.0000
	$\alpha_1$	0.2529	0.0135	18.792	0.0000
	$\beta_1$	0.7007	0.0074	94.225	0.0000
	$\gamma$	0.0421	0.0158	2.6605	0.0078
	$\alpha_1 + \beta_1$	0.9536			
APARCH (1,1) **	$\mu$	0.0738	0.0228	3.2380	0.0012
	$\omega$	0.1987	0.0064	31.008	0.0000
	$\alpha_1$	0.2082	0.0062	33.534	0.0000
	$\beta_1$	0.7839	0.0056	140.79	0.0000
	$\gamma$	0.0348	0.0209	1.6668	0.0056
	$\delta$	1.2493	0.0381	32.788	0.0000
	$\alpha_1 + \beta_1$	0.9921			

**Note:** \*\* means that all estimated parameters are significant at 1% level.

**Table 6: Volatility mean reversion rates and half lives**

Model	Mean reversion rate ( $\alpha_1 + \beta_1$ )	Volatility Half-life	Remark
GARCH (1,1)	0.975089	28 days	Stationary
TGARCH (1,1)	0.953635	15 days	Stationary
PGARCH (1,1)	0.992146	88 days	Stationary

**Mean reversion and half-life of volatility**

Two tests are applied to test for volatility mean reversion in this study. The first is the unit root test presented in Table 2. The ADF unit root test result revealed that the return series used in this study are stationary. A stationary series is mean reverting indicating that the volatility of the series finally reverts to its long-run average. Results of volatility mean reversion of the daily returns using GARCH model are reported in Table 6.

Results of the stationary GARCH (1,1) model indicates that volatility mean reversion rate ( $\alpha_1 + \beta_1$ ) which is usually close to one for most financial series is almost fulfilled with  $\alpha_1 + \beta_1 = 0.975089$  for standard GARCH (1,1),  $\alpha_1 + \beta_1 = 0.953635$  for TGARCH (1,1) and  $\alpha_1 + \beta_1 = 0.992146$  for PGARCH (1,1).

The result of half-life of volatility shock estimated by  $L_{half} = \ln \left[ \frac{0.5}{\alpha_1 + \beta_1} \right]$  to measure the average number of time periods it takes the volatility to revert to its long run average is presented in Table 6. When the value of  $\alpha_1 + \beta_1$  is close to 1, the half-life of a volatility shock is longer. If  $(\alpha_1 + \beta_1) > 1$ , the GARCH model is said to be non-stationary and the volatility eventually explodes to infinity, and the series will follow a random walk, (Goudarzi & Ramanarayanan, 2010). Results of estimated GARCH models of the volatility half-lives are 28 days (approximately one month) for basic GARCH (1,1) model, 15 days ( half a month) for TGARCH (1,1) model and 88 days (approximately three months) for PGARCH (1,1) model. Thus, the return series under review is stationary and mean reverting. As policy implication for investors, stationary and mean reverting asset returns are better options for long term investment.

To test for the presence of ARCH effects in the estimated models, a heteroskedasticity test is performed on the residuals of the estimated models. The result is presented in Table 7.

**Table 7: Heteroskedasticity test for residuals of the estimated models**

Model	Lag Order	F-statistic	P-value	nR <sup>2</sup>	P-value
GARCH (1,1)	1	0.005173	0.9427	0.00518	0.9427
	31	0.241406	1.0000	7.52125	1.0000
TGARCH (1,1)	1	0.486310	0.4856	0.48646	0.4855
	31	0.317737	0.9999	9.89461	0.9999
APARCH (1,1)	1	0.084209	0.7717	0.08424	0.7716
	31	0.328909	0.9998	10.2418	0.9998

The results of our ARCH LM test show that the estimated models have captured all the ARCH effects up to lag 31 and none is remaining in the residuals. This is justified by the p-values of the F-statistics and nR<sup>2</sup> which are statistically insignificant. This shows that our estimated GARCH-type models are good, adequate, valid and accurate in describing the volatility situation in Nigeria.

**Conclusions**

This paper has attempted to model the volatility of asset returns with stylized facts using the Guinness bottling Company Plc daily closing share prices of the Nigerian stock exchange as proxy for Nigerian stock market. The data used for the study covers the period 1/02/1995 to 24/11/2014. The volatility of asset returns was modeled using symmetric standard GARCH (1,1), asymmetric TGARCH (1,1) and PGARCH (1,1) models with Gaussian errors. The result of the standard GARCH (1,1) model showed evidence of volatility clustering (the tendency of large changes in returns to be followed by large changes and small changes to be followed by small changes) and mean reversion in Nigerian stock market. The conditional volatility was found to be quite persistence. The estimated asymmetric TGARCH (1,1) and PGARCH (1,1) models produced supportive evidence for the existence of asymmetry and leverage effects in Nigerian stock market. The study also found that the log returns are non-Gaussian, leptokurtic, fat-tailed and serially uncorrelated. However, the absolute returns were found to be serially correlated. The study therefore concludes that the well-documented stylized facts found in advanced and developed stock markets are also present in emerging stock markets like Nigeria. The study thus recommends both local and foreign traders and investors to invest heavily in Guinness Plc as stable and mean reverting asset returns are less risky.

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